Robots and People

Vlad Estivill-Castro (2016)

Robots for People

--- A project for intelligent integrated systems

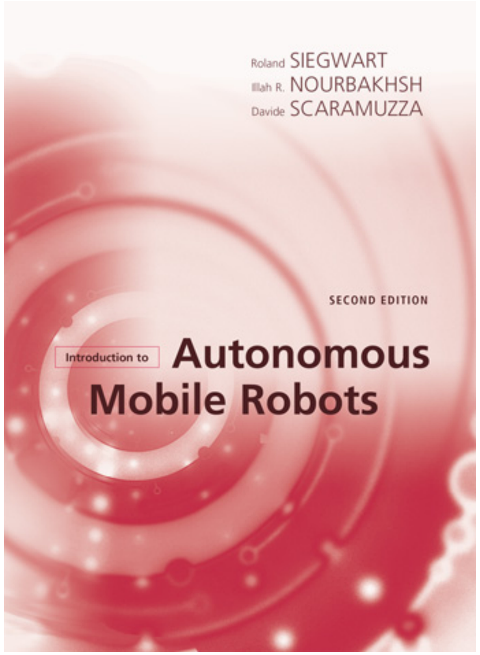
Kinematic Concepts

Lets move the robot from x to y

Chapter 3 (textbook)

What is the course about?

- textbook
- **Introduction to Autonomous Robots**



- second edition

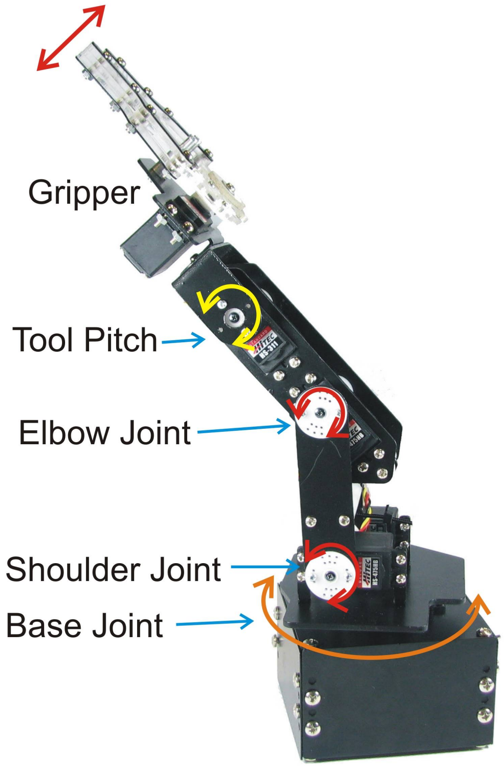
  Roland Siegwart, Illah R. Nourbakhsh, and Davide Scaramuzza

Mobile robot kinematics

- Mobile robot and manipulator arm characteristics
  - Arm is fixed to the ground and usually comprised of a single chain of actuated links

CONTARST with

  - Mobile robot motion is defined through rolling and sliding constraints taking effect at the wheel-ground contact points



Kinematics

- From *kinein* (Greek); to move
  Kinematics is the subfield of Mechanics which deals with motions of bodies

Manipulator- vs. Mobile Robot Kinematics

- Both are concerned **with forward and inverse kinematics**
- **However, for mobile robots, encoder values DO NOT map to unique robot poses.**
- Moreover, **mobile robots** can move unbound with respect to their environment
  - There is **no direct** (=instantaneous) way to measure the robot's position
  - **Position must be integrated over time**, depends on path taken
  - Leads to inaccuracies of the position (motion) estimate
- Understanding mobile robot motion starts with **understanding wheel constraints**

Non-Holonomic Systems

- differential equations are not integrable to the final position.
- the measure of the traveled distance of each wheel is not sufficient to calculate the final position of the robot. One has also to know how this movement was executed as a function of time.
- This is in stark contrast to actuator arms

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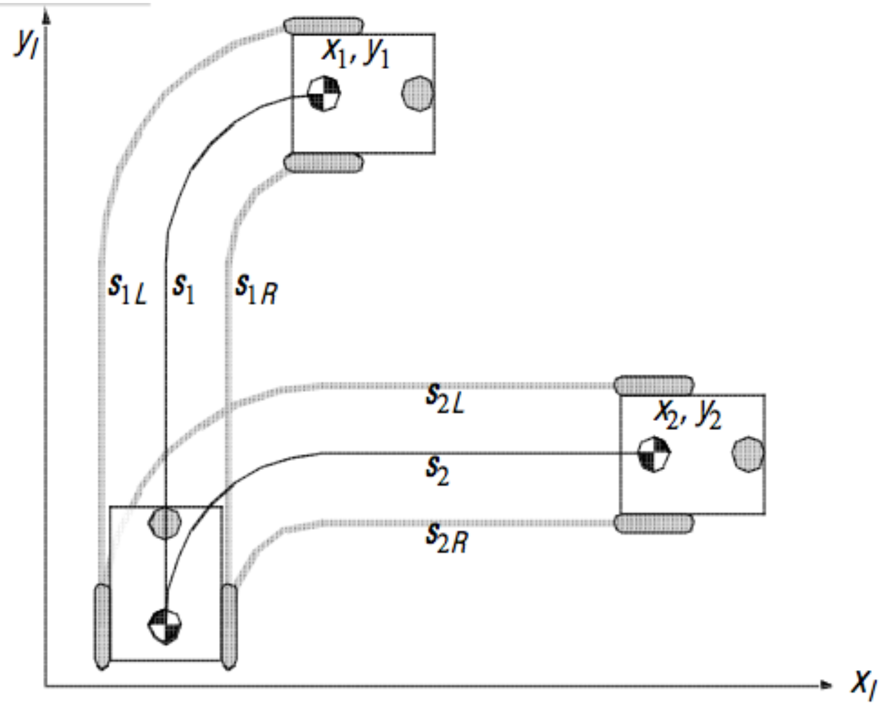
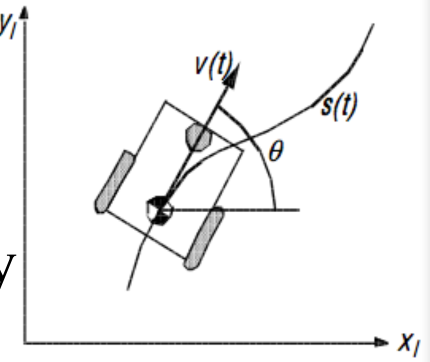
Illustration

$$s_1 = s_2, s_{1R} = s_{2R}, s_{1L} = s_{2L}$$

$$x_1 \neq x_2, y_1 \neq y_2$$



Holonomic system



- A mobile robot is running along a trajectory *s(t)*.
  - At every instant of the movement its velocity *v(t)* is:

$$v(t) = \frac{\partial s}{\partial t} = \frac{\partial x}{\partial t}\cos\theta + \frac{\partial y}{\partial t}\sin\theta$$

$$ds = dx\cos\theta + dy\sin\theta$$

- Function *v(t)* is said to be integrable (holonomic) if there exists a trajectory function *s(t)* that can be described by the values *x*, *y*, and θ only.
- *s=s(x,y,θ)*
- This is the case if

  see book(proof)

$$\boxed{\frac{\partial^2 s}{\partial x \partial y} = \frac{\partial^2 s}{\partial y \partial x} \;;\; \frac{\partial^2 s}{\partial x \partial \theta} = \frac{\partial^2 s}{\partial \theta \partial x} \;;\; \frac{\partial^2 s}{\partial y \partial \theta} = \frac{\partial^2 s}{\partial \theta \partial y}}$$

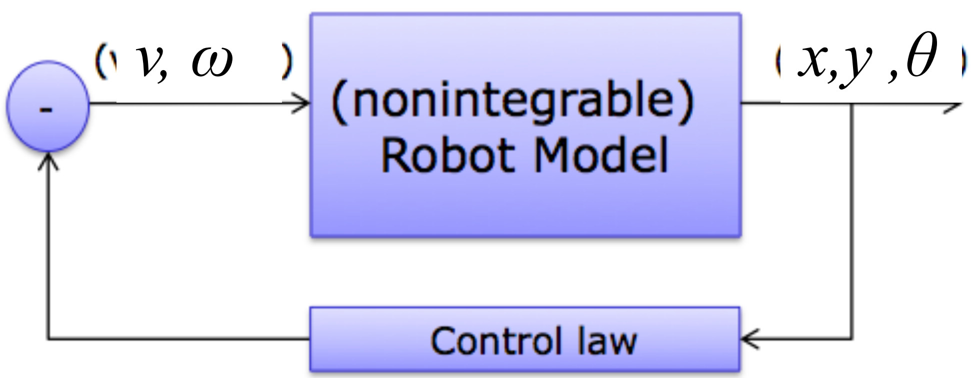
- Condition for *s* to be integrable function

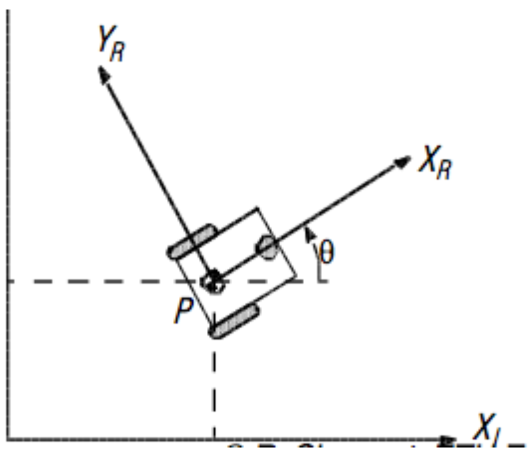
Forward and Inverse Kinematics

- Forward kinematics:
  - Transformation from joint- to physical space
    - robots has angle α in first join and angle β in second join, thus the end of the arm is at x,y and pointing with orientation θ
- Inverse kinematics
  - Transformation from physical- to joint space
    - We have a final position (x,y,θ) of the robot in space, what shall be the angle α of the first join and the angle β of the second join to place the robot there
  - Required for motion control

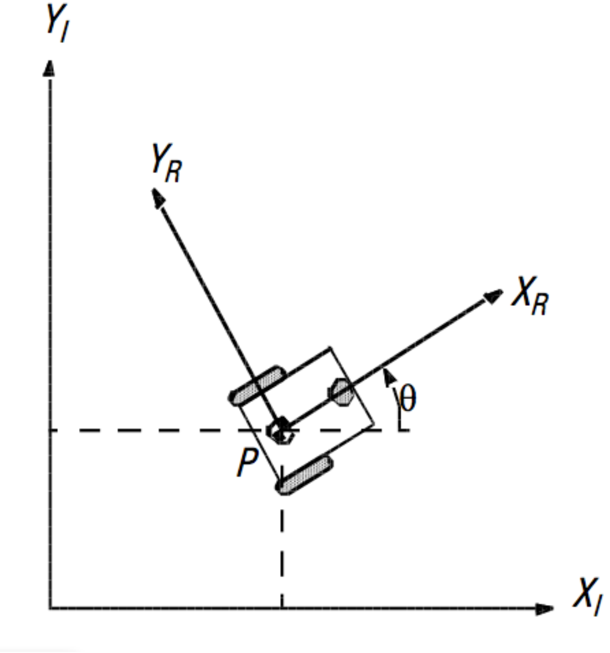
Differential (inverse) kinematics

- Needed for nonholonomic constraints of mobile robotics
  - transform between velocities instead of positions
- Differential kinematic model of a robot
  - $v,\ \omega$ forward and angular velocities





Representing robot pose



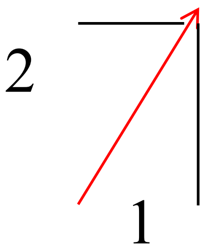
- Arbitrary initial frame
  - Inertial frame: $\{X_I, Y_I\}$
  - Robot frame: $\{X_R, Y_R\}$
  - Robot pose: $\xi_I = [x\ y\ \theta]^T$
    - θ difference between frames
  - Mapping between the two frames

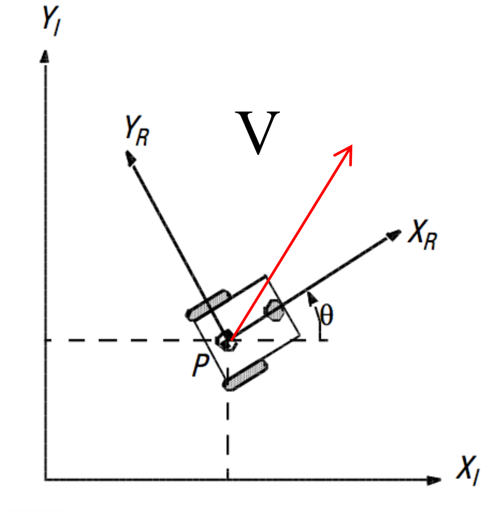
(warning, velocities)

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I = R(\theta)\cdot\begin{bmatrix} x & y & \dot{\theta} \end{bmatrix}^T$$

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example





- $X_I=2, Y_I=1$
- $\theta=30$ $\cos\theta = 0.86$ $\sin\theta=0.5$
- R(30)=
  - [0.86 0.5 0]
  - [-0.5 0.86 0]
  - [0 0 1]
- Given some velocity vector in global frame
  - $(vx_I, vy_I, v\theta_I)$=(1,2,30) it becomes
  - (1.86,0.36,30)

Forward Kinematic Models

- Easy for the differential robot
  - *P* is on the axel between the two wheels
  - $\xi_I = f$ (wheel speeds, steering angles, steering speeds)
    - *Controlling wheel speeds, steering angles and steering speeds, place the robot in global frame*
  - $\xi_I = \mathrm{R}(\theta)^{-1} \xi_R$
  - Find the components of ξ_R as a function of the parameters
    - If we move one wheel forward and the other stationary, *P* moves forwards at half the speed
    - If one moves forward and the other backwards, *P* is fixed.
    - Neither wheel can contribute to a sideway motion

Differential wheel robot

- φ_1 speed right wheel, φ_2 sped of left wheel; r diameter of wheels, s half separation of wheels
- $x_R = r\,\varphi_1/2 + r\,\varphi_2/2$
- $y_R = 0$
- Change in rotational velocity ω_1 when only forward spin of right wheel
  - robot moves clockwise $\omega_1 = r\,\varphi_1/2s$
- *Change in rotational velocity ω_2 when only the left wheel moves forward*
  - *robot moves counterclockwise* $\omega_2 = - r\,\varphi_2/2s$
  - Total rotational velocity $r\,\varphi_1/2s - r\,\varphi_2/2s$

Robots and people

Complete kinematic model

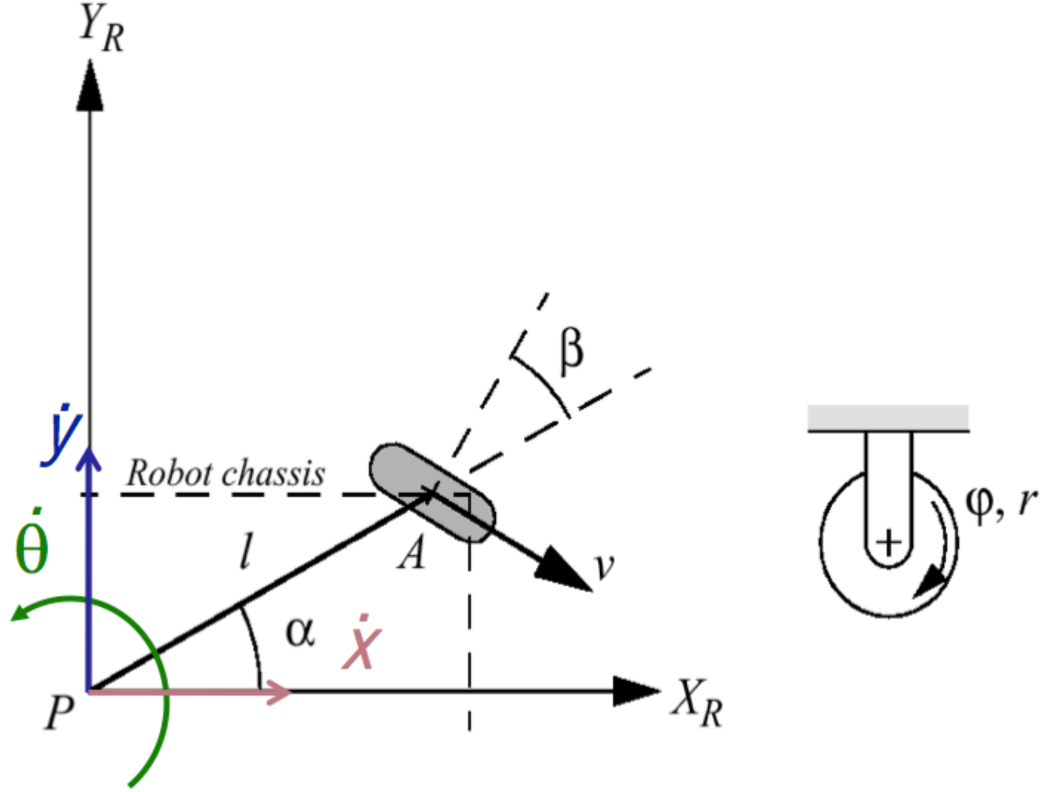
- **Differential wheel robot**
- $\xi_I = R(\theta)^{-1}\, \xi_R$
- $\xi_I = R(\theta)^{-1}\, [r\, \varphi_1/2 + r\, \varphi_2/2\ ,\ 0\ ,\ r\, \varphi_1/2s - r\, \varphi_2/2s]$
  - *NOTE: vertical vector*
- $R(\theta)^{-1} = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{vmatrix}$

More challenging exercise for other robots /wheels/ arrangements

- Wheel kinematic constraints
  - Assumptions
    - Movement on a horizontal plane
    - Point contact of the wheels
    - Wheels not deformable
    - Pure rolling ($v_c = 0$ at contact point)
    - No slipping, skidding or sliding
    - No friction for rotation around contact point
    - Steering axes orthogonal to the surface
    - Wheels connected by rigid frame (chassis)

Fixed Standard Wheel

- l distance, α angle (polar coordinates of where the wheel in robot’s reference frame)
- β angle relative to chassis, *r* radius, *φ(t)* rotation as a function of time



Kinematic Constraints: Fixed Standard wheel

- There is pure rolling at the contact point

$$\left[\sin(\alpha+\beta)\ \ -\cos(\alpha+\beta)\ \ (-l)\cos\beta\right]R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

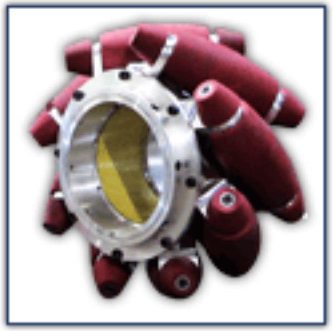
- Sliding constraint, motion orthogonal to wheel plane is zero

$$\left[\cos(\alpha+\beta)\ \ \sin(\alpha+\beta)\ \ l\sin\beta\right]R(\theta)\dot{\xi}_I = 0$$

- SIMPLER if $\alpha=0$ and $\beta=0$
- Robot placed in $X_l = 0$ and $X_l=X_r$, so no sliding sideways

More complex kinematic model for other wheels but no kinematic constraints

- Check textbook and textbook lecture notes for
  - Castro wheel
  - Swedish wheel
  - Spherical wheel



Complete robot

- Given a robot with M wheels
  - each wheel imposes zero or more constraints on the robot motion
  - **only fixed and steerable standard wheels impose constraints**
- What is the maneuverability of a robot considering a combination of different wheels?
- Suppose we have a total of $N=N_f+N_s$ standard wheels
  - N_f number of fixed wheel, N_s number of steerable
  - We can develop the equations for the constraints in matrix forms:

Complete robot

- We can develop the equations for the constraints in matrix forms:
  - on the same plane
    - Rolling

$$J_1(\beta_s)R(\theta)\dot{\xi}_I + J_2\dot{\varphi} = 0 \qquad \varphi(t) = \begin{bmatrix} \varphi_f(t) \\ \varphi_s(t) \end{bmatrix}_{(N_f+N_s)\times 1} \qquad J_1(\beta_s) = \begin{bmatrix} J_{1f} \\ J_{1s}(\beta_s) \end{bmatrix}_{(N_f+N_s)\times 3} \qquad J_2 = diag(r_1 \cdots r_N)$$

    - Lateral movement

$$C_1(\beta_s)R(\theta)\dot{\xi}_I = 0 \qquad C_1(\beta_s) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}_{(N_f+N_s)\times 3}$$

  - in the case of differential wheel robot reduces to the analysis made before

Mobile Robot Maneuverability

- The maneuverability of a mobile robot is the combination of
  - the mobility available based on the sliding constraints
  - plus additional freedom contributed by the steering
- Three wheels is sufficient for static stability
  - additional wheels need to be synchronized
  - this is also the case for some arrangements with three wheels
- It can be derived using the equation seen before
  - Degree of mobility δ_m
  - Degree of steerability δ_s
  - Robots maneuverability $\delta_M = \delta_m + \delta_s$

Degree of Mobility

- To avoid any lateral slip, the motion vector $R(\theta)\ \xi_I$ has to satisfy the following constraints:

$$C_{1f} R(\theta)\dot{\xi}_I = 0$$

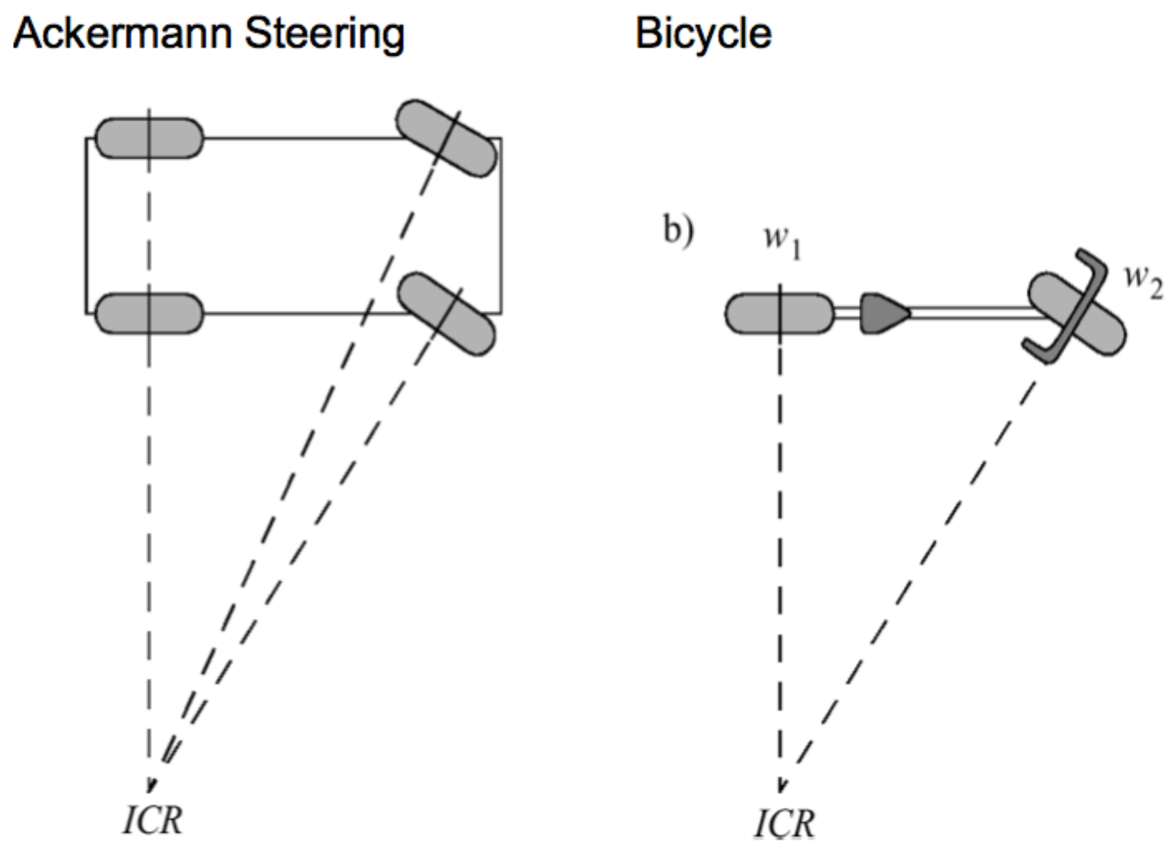
$$C_{1s}(\beta_s) R(\theta)\dot{\xi}_I = 0$$

$$C_1(\beta_s) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}$$

- Mathematically
  - $R(\theta)\ \xi_I$ must belong to the null space of the projection matrix $C_l(\beta_s)$
    - The nulls pace of $C_l(\beta_s)$ is the space N of vectors that are orthogonal, ie, the n so that
      - $C_l(\beta_s)\ n = 0$
- Geometrically, this can be shown by the Instantaneous Center of Rotation (ICR)

Instantaneous Center of Rotation ICR

- The axes of rotation (must intersect the center of radius of movement of the wheels)



Degree of Mobility

- Robot chassis kinematics is a function of the set of independent constraints
  - the greater the number of independent constraints, the greater the rank of $C_l(\beta_s)$, the more constrained is the mobility
- Examples
  - (unicycle) one standard wheel => rank 1, single independent constraint on mobility
  - differential drive, same axel => rank 1 (but two constraints)
  - bicycle, same plane => rank 2 (two independent constraints)
  - if rank$[C_l(\beta_s)] > 1$ the vehicle can only travel on circle or along a straight line
  - rank $[C_l(\beta_s)] = 3$ motion is impossible, all directions constrained
  - rank $[C_l(\beta_s)] = 0$ no standard wheels

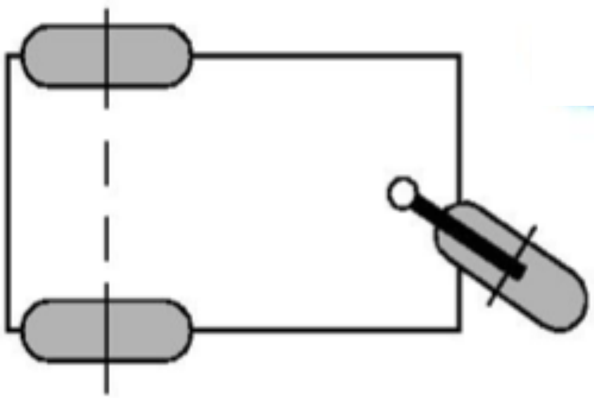
Degree of Steerability

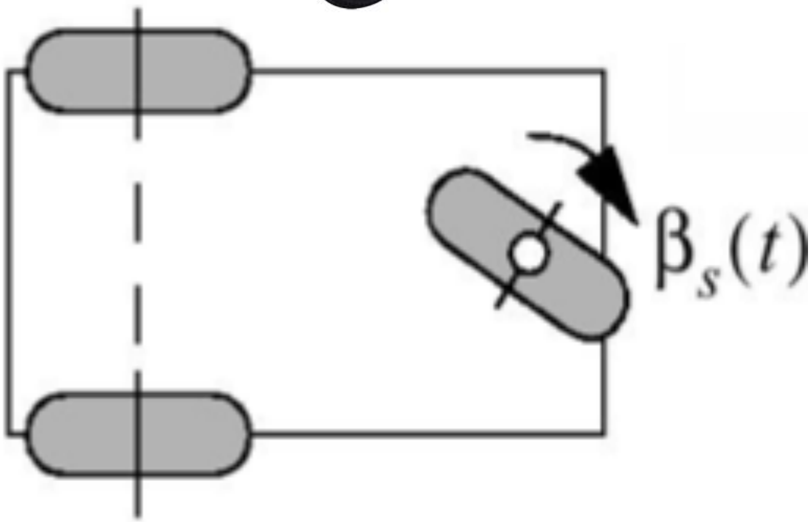
- Indirect degree of motion
  - $\delta_s = \text{rank}\ [C_1(\beta_s)]$
  - The particular orientation at any instant imposes a kinematic constraint
  - However, the ability to change that orientation can lead additional degree of maneuverability
  - range of δ_s : $0 \leq \delta_s \leq 2$
- *Examples*
  - one steered wheel: Tricycle
  - two steered wheels: No fixed standard wheel
    - car (Ackermann steering): $N_f = 2$, $N_s=2$
      - IMPLIES common axle

Degree of Maneuverability

- $\delta_M = \delta_m + \delta_s$
- Two robots with same δ_M are not necessary equal
- Example:
  - Differential drive and Tricycle (next slide)
- For any robot with $\delta_M = 2$ the ICR is always constrained to *lie on a line*
- For any robot with $\delta_M = 3$ the ICR is not constrained and can *be set to any point on the plane*
- Example
  - The Synchro Drive example: $\delta_M = \delta_m + \delta_s = 1+1=2$
  - Can *move in any direction, but the orientation is not controllable*

Wheel configurations

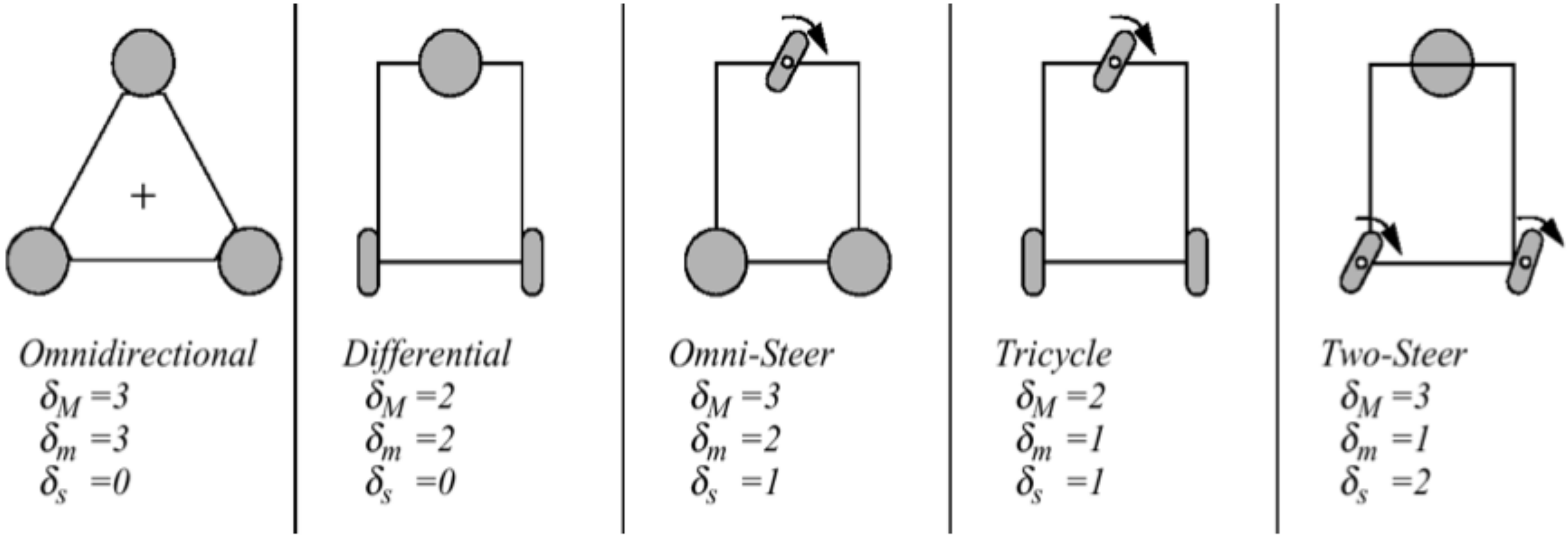




Differential drive

Tricycle

Five basic types of 3-wheel configurations



Synchro Drive

- $\delta_M = \delta_m + \delta_s = 1 + 1 = 2$
- **LEGO Mindstorms NXT, HOLO3CYCLE**
  - http://www.youtube.com/watch?v=dqPMegizjwk&feature=endscreen
- **Lego Synchro Drive**
  - **http://www.youtube.com/watch?v=MFxjIthqXVs**

Mobile Robot Workspace

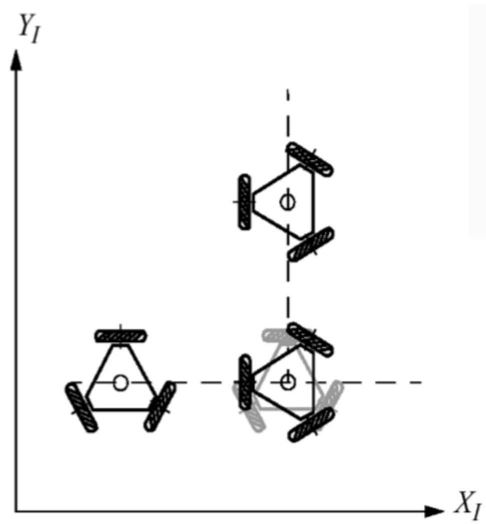
- The degrees of Freedom (DOF) is the robot's ability to achieve various poses.
- But what is the degree of vehicle's freedom in its environment
  - Car example
- Workspace
  - how the vehicle is able to move between different configuration in its workspace?
- The robot's independent achievable velocities
- Differentiable degrees of freedom (DDOF) $=\delta_m$
  - bicycle : $\delta_M = \delta_m + \delta_s = 1+1=2$ DDOF $=1$ DOF$=3$
  - omni drive : : $\delta_M = \delta_m + \delta_s = 3+0$ DDOF$=3$ DOF$=3$

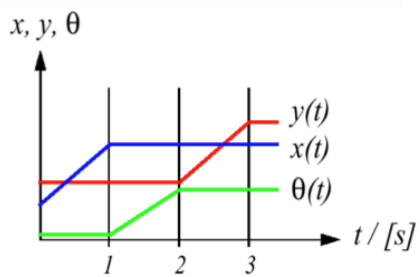
Degrees of Freedom, Holonomy

- DOF *degrees of freedom*:
  - Robots ability to achieve various poses
- DDOF *differentiable degrees of freedom*:
  - Robots ability to achieve various trajectories
  - DDOF $\leq \delta_M \leq$ DOF
- Holonomic Robots
  - A holonomic kinematic constraint can be expressed as an explicit function of position variables only
  - A non-holonomic constraint requires a different relationship, such as the derivative of a position variable
  - *Fixed and steered standard wheels impose non-holonomic constraints*

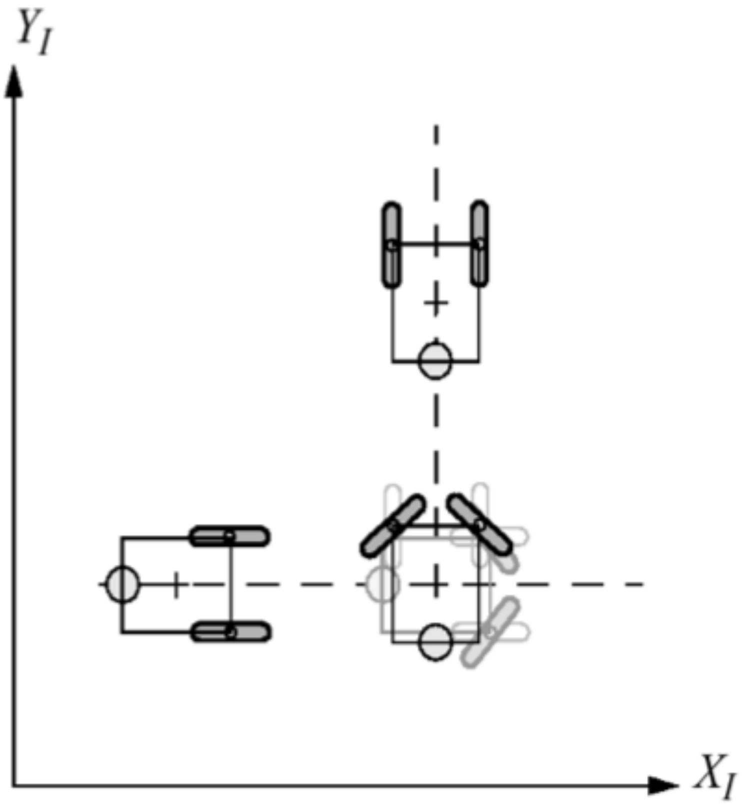
Omnidirectional drive

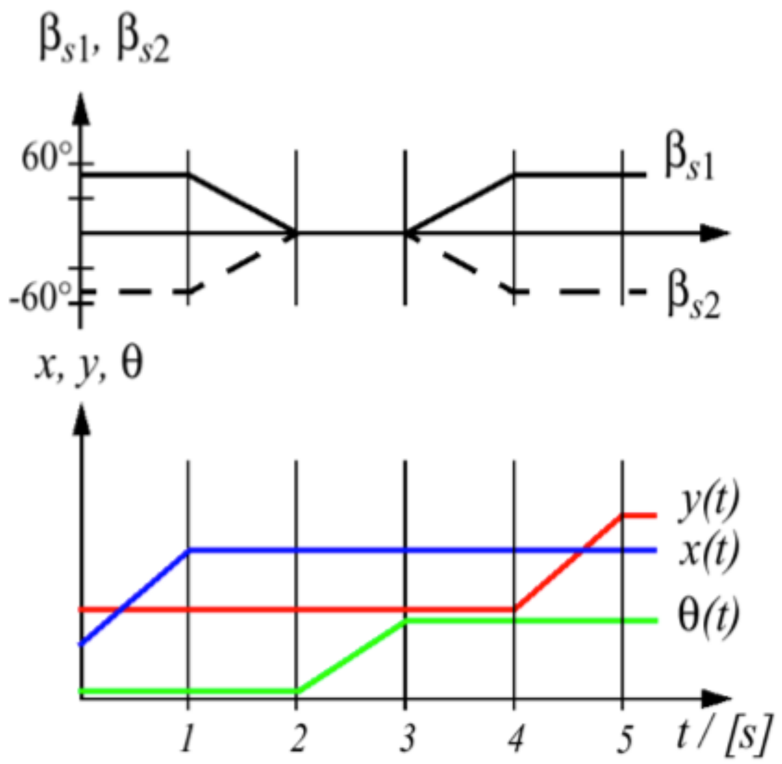
- Difference between path and trajectory from P_1 to P_2
  - Omni directional drive achieves any trajectory
    - what the motors do, may be the challenge of kinematics





Another example two steer





Beyond classical kinematics

- Dynamics:
  - High speeds, difficult terrains
  - No-sliding does not hold

Robot Motion Control

Overview

Robot Motion Control

- The objective of a kinematic controller is to follow a trajectory described by its position and/or velocity profiles as function of time.
- Motion control is not straight forward because mobile robots are typically non-holonomic and MIMO systems.
- Most controllers (including the one presented here) are not considering the dynamics of the system

Open Loop Control

- trajectory (path) divided in motion segments of clearly defined shape:
  - straight lines and segments of a circle
    - Dubins car, and Reeds-Shepp car
  - control problem:
    pre-compute a smooth trajectory based on line, circle (and clothoid) segments
- Disadvantages:
  - It is not at all an easy task to pre-compute a feasible trajectory
  - limitations and constraints of the robots velocities and accelerations
  - does not adapt or correct the trajectory if dynamical changes of the environment occur.
  - The resulting trajectories are usually not smooth (in acceleration, jerk, etc.)

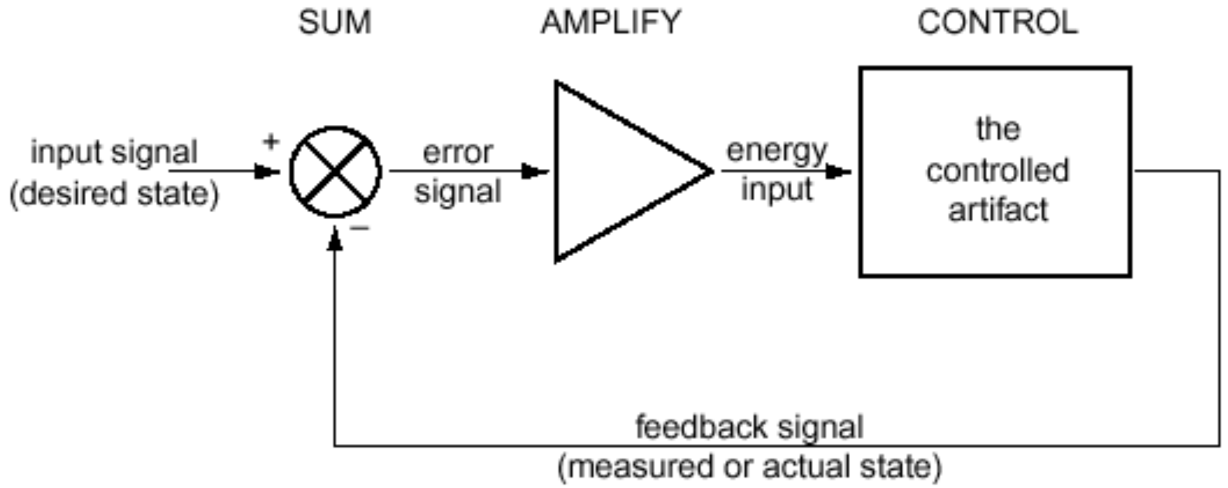
Open Loop Control

- Does not use sensory feedback, and state is not fed back into the system
- Feed-forward control
  - The command signal is a function of some parameters measured in advance
  - E.g.: **battery strength** measurement could be used to **"predict"** how much **time** is needed for the turn
  - Still open loop control, but a computation is made to make the control more accurate
- Feed-forward systems are effective only if
  - They are well calibrated
  - The environment is predictable & does not change such as to affect their performance

Robots and people

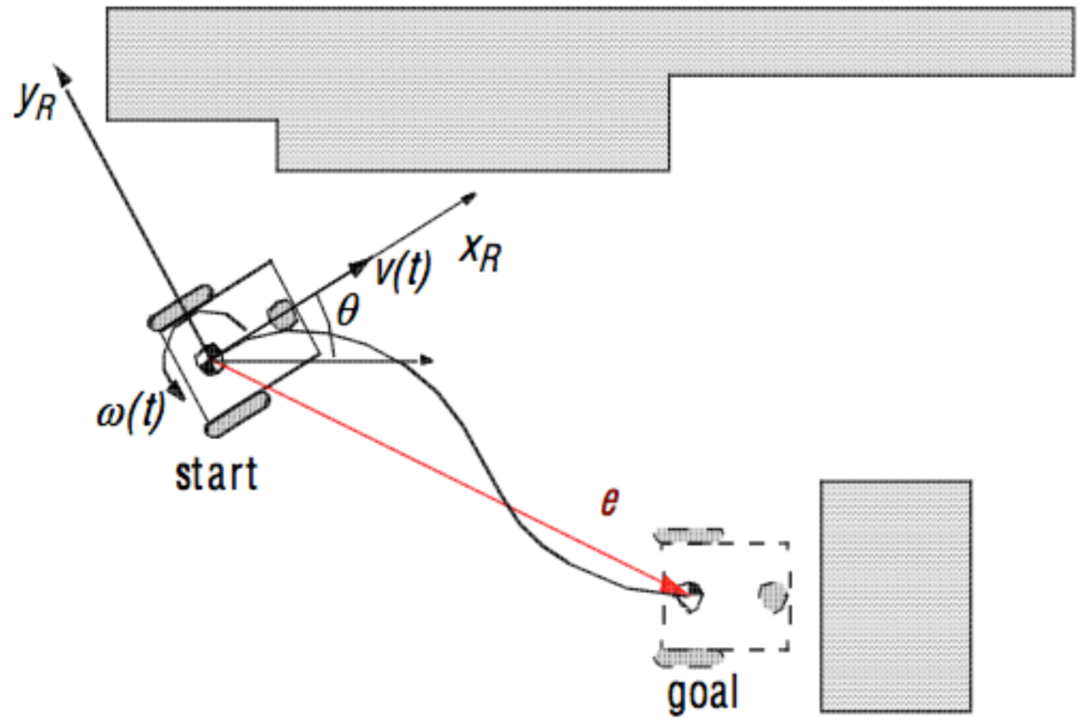
Feedback Control

- Feedback control = having a system **achieve** and **maintain** a **desired state** by continuously comparing its current and desired states, then adjusting the current state to minimize the difference
- Also called closed loop control



Motion Control: Feedback Control

- Problem statement: From start position, get to the goal position (in reference frame of the robot)



Formalization

- The control elements are the velocity $v(t)$ in x_R and the angular velocity $\omega(t)$
- The error vector in the coordinates of the robot is $e=(x,y,\theta)$

- FIND a control matrix K (if it exists) such that $K\ e$ drives the error to zero

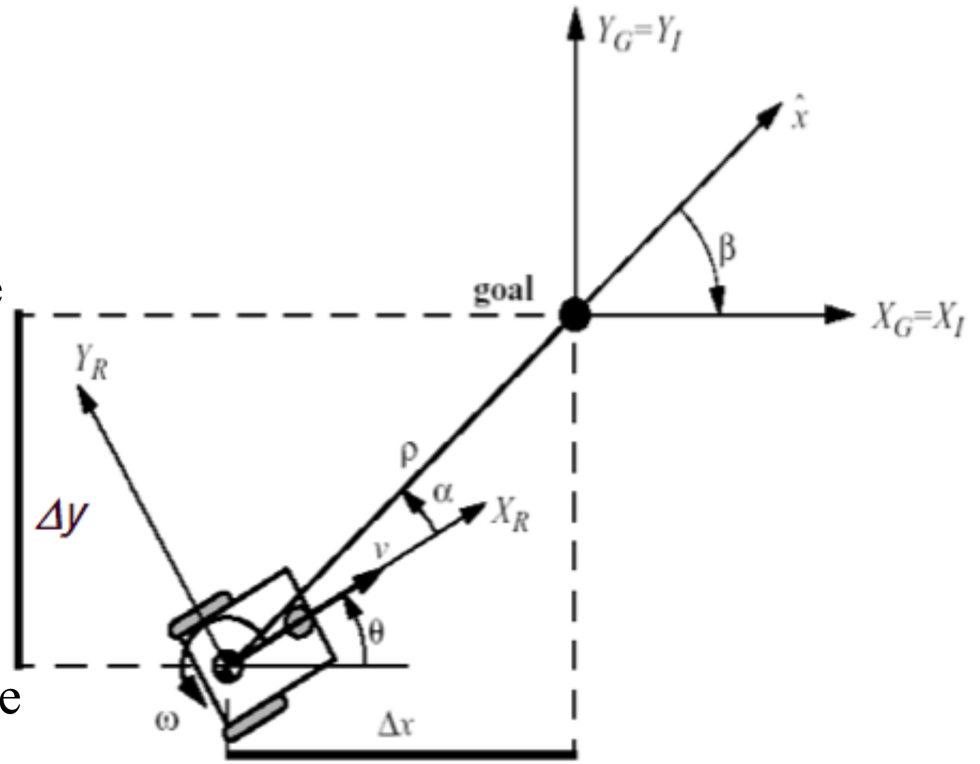
Kinematic Position Control

The kinematics of a differential drive mobile robot described in the inertial frame $\{x_I, y_I, \theta\}$ is given by,

$$
{}^{I}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}
$$

where x and y are the linear velocities in the direction of the x_I and y_I of the inertial frame.

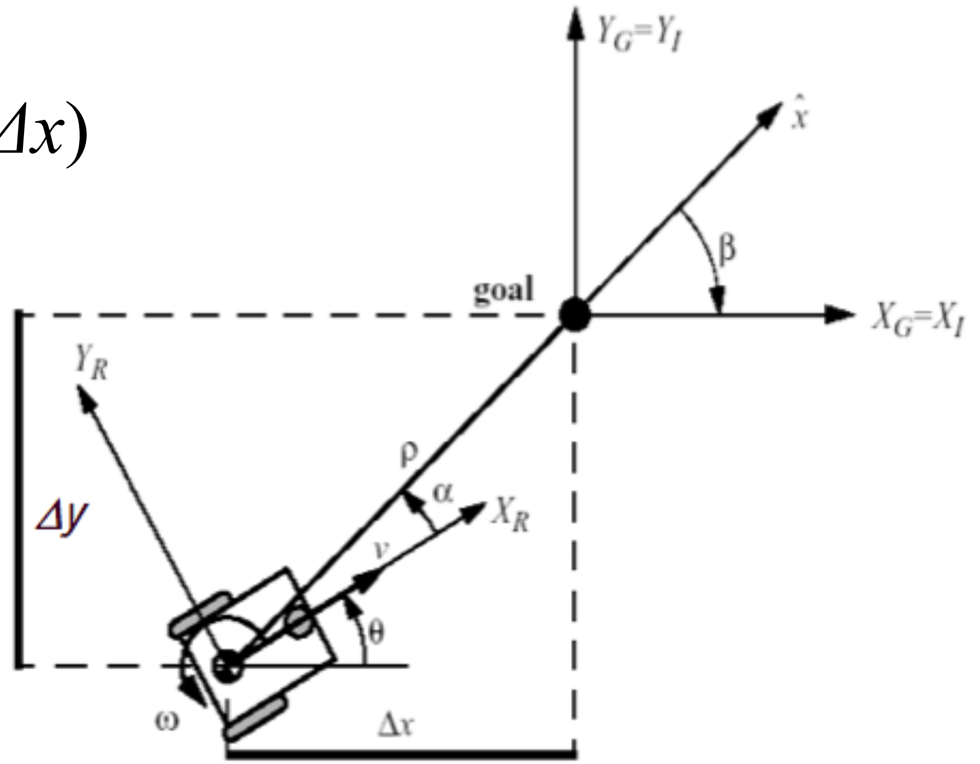
Let α denote the angle between the x_R axis of the robots reference frame and the vector connecting the center of the axle of the wheels with the final position.



Coordinate transformations

- Coordinates transformation into polar coordinates with its origin at goal position:
  - $\rho=\sqrt{(\Delta x^2+\Delta y^2)}$
  - $\alpha=-\theta+\operatorname{atan2}(\Delta y,\Delta x)$
  - $\beta=-\theta-\alpha$

If the robot moves forward at velocity v the change in ρ is $v\cos\alpha$



Robots and people

System description

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

- *α in $(-\pi/2 , \pi/2)$=Interval$_1$*

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\cos\alpha & 0 \\ \dfrac{\sin\alpha}{\rho} & -1 \\ -\dfrac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

- *α in $(-\pi/,- \pi/2) \vee (\pi/2 , \pi)$=Interval$_2$*

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 \\ -\dfrac{\sin\alpha}{\rho} & -1 \\ \dfrac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

Remarks

- The coordinate transformations *is* not defined at $x=y=0$;
- For α *in* $Interval_1$, the forward direction of the robot points towards the goal, for α *in* $Interval_2$, it is the backward direction
- By properly defining the forward direction of the robot at its initial configuration, it is always possible to have α *in* $Interval_1$ at $t=0$. However this does not mean that α remains in $Interval_1$ for all time t.

The Control Law

- Proportional derivative control
  - $v = k_\rho \rho$ *and* $\omega = k_\alpha \alpha + k_\beta \beta$
  - the feedback control system will drive the robot to $(\rho, \alpha, \beta) = (0,0,0)$

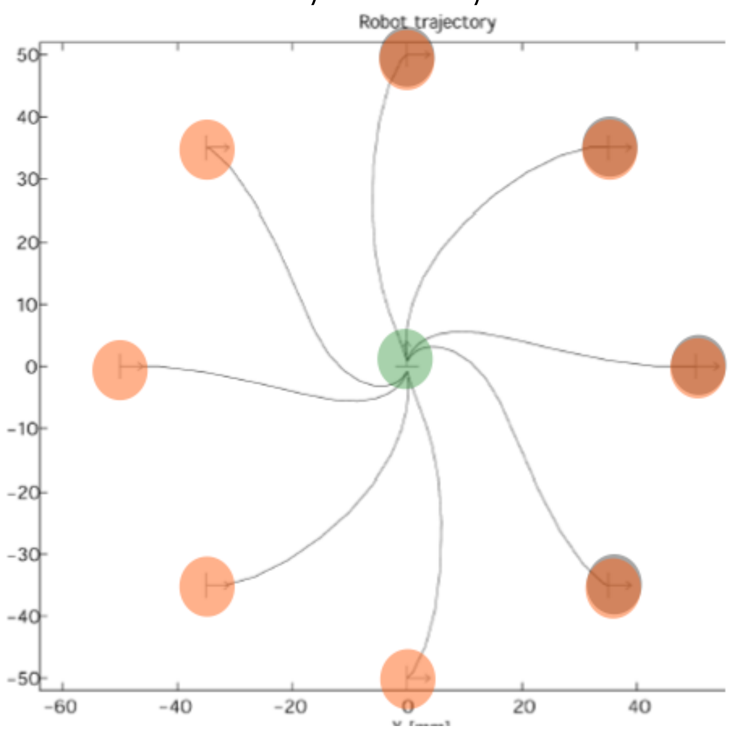
$$\begin{bmatrix} \dot\rho \\ \dot\alpha \\ \dot\beta \end{bmatrix} = \begin{bmatrix} -\cos\alpha & 0 \\ \frac{\sin\alpha}{\rho} & -1 \\ -\frac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \qquad \begin{bmatrix} \dot\rho \\ \dot\alpha \\ \dot\beta \end{bmatrix} = \begin{bmatrix} -k_\rho \rho \cos\alpha \\ k_\rho \sin\alpha - k_\alpha \alpha - k_\beta \beta \\ -k_\rho \sin\alpha \end{bmatrix}$$

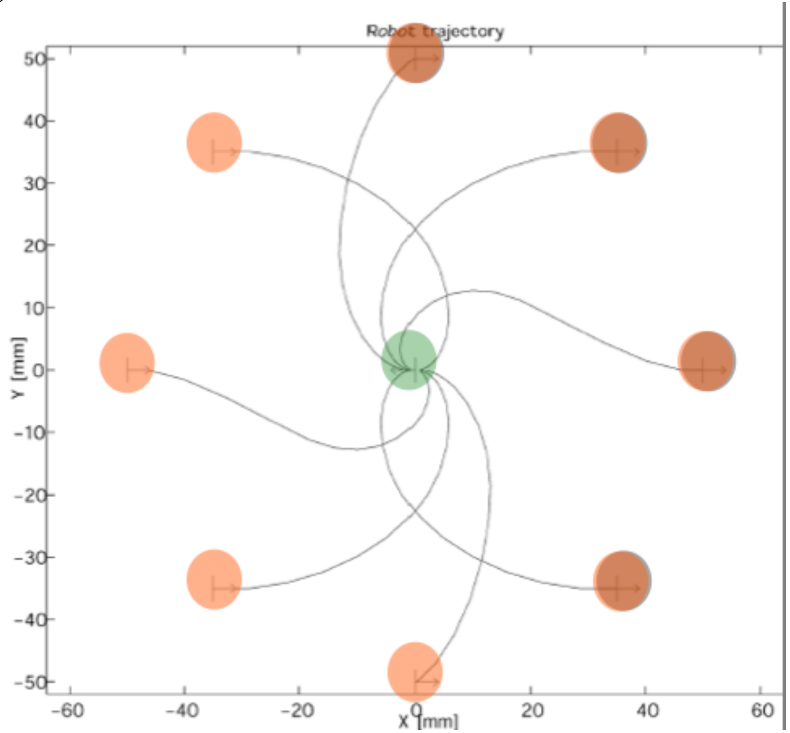
  - The control signal v(t) has always constant sign

Robots and People

Very natural ways to perform maneuvers *(like parking-- without ever inverting motion)*

- The goal is in the center and initial position on the circle
  - the angle is 90° different
  - the parameters of the feedback-control is
    - $k=(k_\rho , k_\alpha , k_\beta)=(3,8,-1.5)$





Summary

- Kinematics – move the robot from known position to new position
- Models can be challenging for non-holonomic robots.
- Standard wheels and steering wheels impose constraints.
- There is reasons why some robots have easier kinematic models and kinematics controllers

THANK YOU

Robots and people